Average irregularity representation of a rough surface for ray reflection

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A new ray model is presented for the reflection of electromagnetic radiation from the rough air–material interface of a randomly rough surface. Unlike previous derivations that modeled the rough interface as consisting of microareas randomly oriented but flat (facets), this derivation models it as consisting of microareas not only randomly oriented but also randomly curved. Physically, the models are the same, but this new derivation leads to some new results. (1) For any given rough surface, there exists a single, optically smooth, curved surface of revolution of very restricted shape that will reflect radiation in the same distribution as that reflected by the rough interface. (2) Modeling that surface as an ellipsoid of revolution gives a surface-structure function that appears more accurate and useful than existing ones. (3) Unlike the facet derivations, this derivation lends itself to a normalization that gives the absolute, instead of just a comparative, reflectance-distribution function.

Index Headings: Geometrical optics; Reflection; Scattering.

A theory for single reflection by the rough, air–material interface of a randomly rough surface was first presented by Bouguer.[1] He modeled the interface as consisting of randomly oriented, optically flat facets, each of which behaves like a small plane mirror, that reflects a portion of the light incident on it in the manner determined by Fresnel's equations for specular reflection at a flat dielectric interface.

The Bouguer facet theory has been found[2–6] to explain much of the reflection from rough surfaces and has undergone considerable development. The derivation of the basic theory has been refined[2,3,7–11] and several models[3,4,7,10,12,13] have been tried for the microarea distribution function. This is the function that gives the relative number of facets oriented in any given direction, or, more precisely, the relative total facet surface area per unit solid angle of surface normals pointed in any given direction. This function governs the directional distribution of the scattered light. If the function is uniform with respect to the direction of the facet normals, a diffuse-like distribution occurs. If most of the facets lie near the plane of the surface, a nearly specular distribution occurs.

This paper presents a new model for ray-theory reflection from the rough, air–material interface. Instead of randomly oriented flat microareas (facets), here the surface structure is modeled as an ensemble of randomly oriented and randomly curved microareas. Because for practical purposes, any curved area can be broken down into infinitesimal facets, the two models are identical, physically. However, this new derivation leads to some new results. (i) For any possible (but realistic) rough interface, there exists a single, optically smooth, curved surface of revolution (termed an average surface irregularity) that reflects radiation into the same distribution as that reflected by the rough interface. Furthermore, the shape of the average irregularity can be very greatly restricted and still be general enough to represent any possible (but realistic) rough interface. (ii) This restricted shape is similar to an ellipsoid of revolution. An ellipsoid-of-revolution model for the average irregularity gives a surface-structure function that appears more accurate and useful than existing ones. (iii) Unlike the facet derivations, this derivation lends itself to a normalization that gives the absolute, instead of just a comparative, reflectance-distribution function.

For simplicity and conciseness, we derive a reflectance-distribution function termed[14] the BRIDF, f_{rI} (the bidirectional reflected-intensity-distribution function). It is defined as the intensity reflected into any specified direction from an illuminated point per unit of flux in the incident beam. The more common and more generally useful quantity is termed[15] the BRDF, f_r (the bidirectional reflectance-distribution function). It is defined as the radiance reflected into a given direction per unit of irradiance incident on the surface from another given direction. The BRDF is equal to the BRIDF divided by the cosine of the zenith (or sine of the elevation) angle of reflection.

The paper is organized as follows. First we derive the reflection from a single, curved microarea, then an ensemble of curved microareas gives a reflectance–distribution function and gives it in terms of a surface structure function. The concept of the average irregularity is presented and validated. Next, a normalization procedure makes the reflectance-distribution function an absolute quantity. An ellipsoid of revolution is used as a one-parameter model for the average irregularity. Last, we test the accuracy and usefulness of the resulting surface-structure function for the ellipsoid.

INTERFACE REFLECTION MODEL

Reflection from a single microarea

Consider, as shown in Fig. 1, a very small, curved area ΔA_s on an irregularity of the rough-surface microstructure. This area is chosen small enough so that we

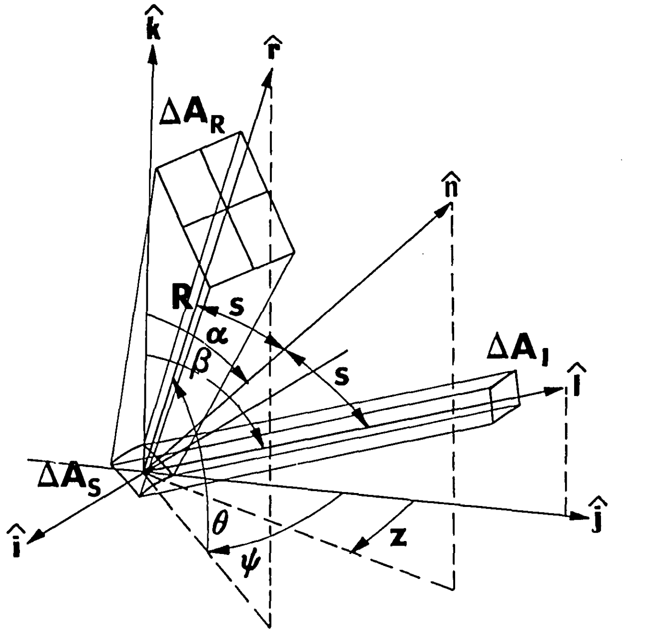

FIG. 1. Reflection from a curved microarea of the rough-surface air–material interface. A small optically smooth, slightly curved area ΔA_s with a normal $\hat{n}$ intercepts the light from an area ΔA_I of the incident beam diverges it into $\Delta\omega = \Delta A_R/R^2$ steradians in a direction $\hat{r}$ such that $\hat{r}\times\hat{n} = \hat{n}\times\hat{l}$.

can approximate the curvatures in all directions on it by sections of circles. (The curvatures of the circle sections may be different in different directions across the area.) Also, the area is chosen small enough so that these circle sections deviate little from straight lines. Then, a single, surface normal $\hat{n}$ specifies the orientation of ΔA_s, and a zenith angle α and an azimuth angle z, defined in Fig. 1, give the direction of $\hat{n}$ relative to the planar macrosurface. Let the incident radiation be in a narrow collimated beam originating from a source in the direction $\hat{l}$, and let it have a flux of Φ_I, spread (not necessarily uniformly) over the beam's cross-sectional area A_I. At the point in A_I, where ΔA_s is located, let the flux density of the incident beam be E_n.

Let ΔA_s be optically smooth and let its minimum dimension and its minimum radius of curvature be much greater than the wavelength of the incident radiation. Then[16,17] ray theory applies, so ΔA_s reflects as a section of a curved mirror surface; that is, of the incident light intercepted by ΔA_s, a portion $\mathcal{R}(s,n,k)$ (determined by the Fresnel equations; n and k are the refraction and absorption indices, respectively) reflects into a small cone (represented for simplicity by the rectangularly cross-sectioned cone in Fig. 1) of solid angle $\Delta\omega$ in a direction $\hat{r}$ determined by the mirror-reflection equation $\hat{r}\times\hat{n} = \hat{n}\times\hat{l}$.

The reflected intensity is derived from the curvatures of ΔA_s as follows. Of the incident-beam cross-sectional area, the area intercepted by ΔA_s is $\Delta A_s \cos s$, so ΔA_s reflects a flux of $\mathcal{R}(s)E_n\Delta A_s\cos s$ into the small, solid angle $\Delta\omega$, giving the reflected intensity $I = \mathcal{R}(s)E_n\Delta A_s\cos s/\Delta\omega$. Let Δl_z (see Fig. 2) be the line segment formed on ΔA_s by the intersection of ΔA_s with a plane parallel to the macrosurface, and let Δl_α be the line segment formed on ΔA_s by the intersection of ΔA_s with the plane containing the macrosurface normal $\hat{k}$ and the ΔA_s-surface normal $\hat{n}$. Because all line segments on ΔA_s deviate only slightly from straight lines, $\Delta A_s = \Delta l_\alpha \Delta l_z$ holds. Because these line segments are approximately sections of circles, they can be given by $\Delta l_z = \sigma_z|\Delta z|$ and $\Delta l_\alpha = \sigma_\alpha|\Delta\alpha|$, where σ_z and σ_α (defined positive) are the radii of curvature of ΔA_s, and Δz and $\Delta\alpha$ are the arcs subtended by Δl_z and Δl_α, respectively. (Strictly, σ_z is not a radius of curvature of ΔA_s because it is not perpendicular to ΔA_s, but this does not affect the derivation.) The reflected solid angle can be given by $\Delta\omega = \cos\theta|\Delta\theta\Delta\psi|$. Substituting the above three expressions into I gives

$$I = \frac{\mathcal{R}(s,n,k)E_n\cos s}{\cos\theta}\left|\frac{\Delta\alpha\Delta z}{\Delta\theta\Delta\psi}\right|\sigma_\alpha\sigma_z \ . \tag{1}$$

Because α and z are functions of θ, ψ, β, the quantity $|\Delta\alpha\Delta z/\Delta\theta\Delta\psi|$ can be given by the jacobian determinant $J(\alpha, z:\theta,\psi)$ (in the limit as the incremental angles approach zero).

Note that the derivation is still valid if either one or both of the lines Δl_z and Δl_α have curvature opposite that shown in Fig. 2. This lets Eq. (1) apply to a curved area of any shape: convex, concave, or saddle. Also, because radii of curvature are defined as positive and the jacobian determinant is used only in an absolute value, I remains unchanged if the curvatures of either Δl_α or Δl_z or both are reversed.

Ensemble of microareas and the surface-structure function

Consider the following model for the surface microstructure. Let the microsurface be continuously curved and randomly undulating, similar to hill or mountain topography. Let it be optically smooth and let all cur-

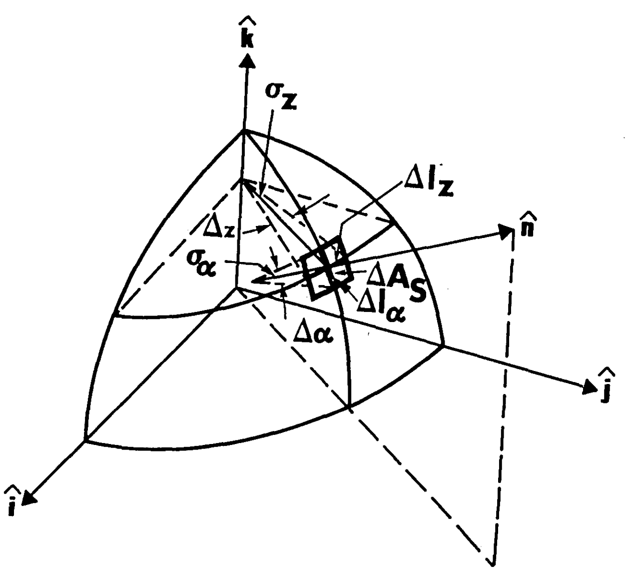

FIG. 2. The shape of a curved microarea in terms of two orthogonal curved lines.

vature radii and irregularity sizes be much larger than the wavelength of the incident radiation. Let the ΔA_s be any small area on this undulating surface. Under these conditions, Eq. (1) gives the reflection from each small area. Equation (1) may be applied to a discontinuous microstructure if we disregard diffraction effects at corners and edges. This allows application of Eq. (1) to a randomly scratched surface, such as ground glass or roughened metal, and to globule-pile-like structure, such as some diffusely reflecting spray-painted surfaces. Also, Eq. (1) may be applied to a surface that contains flat facets if we make their curvature radii not quite infinite. A very wide variety of microstructures is thus included. Finally, let the statistical properties be uniform with position on the surface and independent of direction along the surface (no wood-like grain, or scratches with a preferred direction).

For given directions of illumination and observation, an observer, if his visual resolution were sharp enough to resolve the surface microstructure, would see the reflected light originating only from various points scattered around the microstructure. The microsurface normals at these reflecting points must all be parallel, so the reflection I_i from each point is given by Eq. (1) with the same values of θ, ψ, α, z, and s. If for a particular combination of incidence and reflection directions, we let a given incident beam illuminate N reflecting points, the total intensity reflected in the specified direction is given by the ensemble

$$I_T=\sum_{i=1}^{N} I_i=\sum_{i=1}^{N}\frac{\mathcal{R}(s,n,k)E_{ni}\cos s}{\cos\theta}\left|J\left(\frac{\alpha,z}{\theta,\psi}\right)\right|\sigma_{\alpha i}\sigma_{zi} \;.$$

This equation is simplified by replacing the flux density E_{ni} in the incident beam at each reflecting point by an average value given by $E_{na}=\Phi_I/A_I$, where Φ_I is the flux of the entire incident beam and A_I is the cross-sectional area of the incident beam,

$$I_T=\frac{\mathcal{R}(s,n,k)\Phi_I\cos s}{\cos\theta}\left|J\left(\frac{\alpha,z}{\theta,\psi}\right)\right|\frac{1}{A_I}\sum_{i=1}^{N}\sigma_{\alpha i}\sigma_{zi} \;. \qquad (2)$$

If all dependence on the surface material and laboratory geometry can be separated into one factor and all dependence on the surface structure into a second factor, this second factor would be a surface-structure function. Note that all dependence on the surface structure is localized in the summation factor of Eq. (2); however, this factor contains a dependence on the laboratory geometry; namely, N varies with incidence angle β. This can be removed as follows: For normal incidence and a chosen direction of reflection, consider the set of N_n reflecting points. As the incidence angle increases, this same set of points reflects into a new direction. However, the illuminated macrosurface area increases, as $1/\cos\beta$, and some points in the additional area also contribute to the reflection. So the summation factor for the new intensity must contain some additional $\sigma_{\alpha i}\sigma_{zi}$ quantities. Under the assumption that the surface is statistically uniform, these extra quantities will be statistically the same as the original set, and the summation factor is merely increased by $1/\cos\beta$. Thus Eq. (2) becomes

$$I_T=\frac{\mathcal{R}(s,n,k)\Phi_I\cos s}{\cos\beta\cos\theta}\left|J\left(\frac{\alpha,z}{\theta,\psi}\right)\right|\frac{1}{A_I}\sum_{i=1}^{N_n}\sigma_{\alpha i}\sigma_{zi} \;, \qquad (3)$$

and the incidence-angle dependence has been separated from the summation factor. However, the summation factor still retains a dependence on the laboratory geometry; namely, N_n is proportional to the cross-sectional area A_I of the incident beam. This is easily compensated by including the $1/A_I$ factor with the summation factor.

Thus, the quantity

$$D(\alpha)=\frac{1}{A_I}\sum_{i=1}^{N_n}\sigma_{\alpha i}\sigma_{zi} \quad \text{(dimensionless)} \qquad (4)$$

is a surface-structure function, since it contains all the dependence on the surface structure and no dependence on anything else. In general, this quantity is a function of both the coordinates, α and z, of the microarea normal, but we are considering only surfaces whose statistical properties are directionally uniform. In the Bouguer facet theory and its refinements, the surface-structure dependence is incorporated as a microarea-distribution function, which is the relative amount of microarea oriented in a given direction or the probability density of a facet normal to be in a given direction. Like D, this function contains all the dependence on the surface structure and no dependence on anything else. Since also the two reflectance theories are identical physically, D and microarea distribution function must be related by a function of only α.

Equation (3) can be put in terms of a reflectance-distribution function by dividing by the incident flux Φ_I. This gives the BRIDF.[14] Incorporating Eq. (4) in addition gives

$$f_{rI}(\beta;\theta,\psi)=\mathcal{R}(s,n,k)\frac{\cos s}{\cos\beta\cos\theta}\left|J\left(\frac{\alpha,z}{\theta,\psi}\right)\right|D(\alpha) \;. \qquad (5)$$

(Division by $\sin\theta$ gives the BRDF.[15])

Average surface irregularity

It might prove useful if there could exist a single optically smooth curved surface that would reflect light in the same distribution as that reflected by the ensemble of all the curved microareas comprising the rough surface. This would mean that a randomly irregular surface could be treated as if it consisted of a large number of small identical average irregularities or that a randomly irregular surface could be treated as if it were a single large curved surface (for a "uniform flux density" incident beam of the same total flux). Such an average irregularity would have to be a surface of revolution about the macrosurface normal because of the assumed directional independence of the rough-surface statistics.

Mathematically, the question of the possible existence of such an average irregularity may be stated as follows. Let $\rho_\alpha(\alpha)$ and $\rho_z(\alpha)$ be the radii of curvature for the average irregularity (defined as were, respectively, σ_α and σ_z, for a curved mircoarea), and let C be a constant. Does there exist a surface of revolution whose $D(\alpha)$ given by

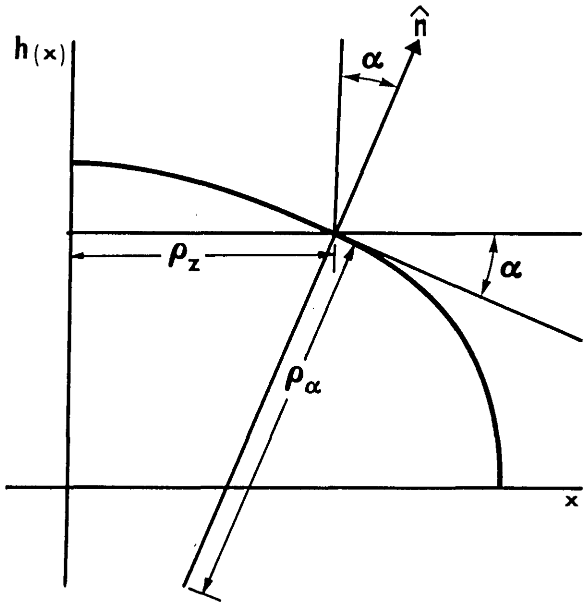

FIG. 3. Cross section of an average surface irregularity. The shape of $h(x)$ may be restricted to the general shape illustrated and still be general enough to represent any microstructure. This general shape consists of the slope being 0 at $x=0$ and infinity at $h=0$ and the curve between having no inflection points or straight-line sections.

$$D(\alpha)=C\rho_\alpha(\alpha)\rho_z(\alpha) \qquad (6)$$

is the same as the $D(\alpha)$ resulting from Eq. (4)? It may be, but is not obvious, that for any realistic functional form of $D(\alpha)$, given by Eq. (4), there exists a surface of revolution which, by Eq. (6), can give this same functional form of $D(\alpha)$. The following proves that this is indeed true and, furthermore, that the shape of the surface of revolution can be very greatly restricted and still be general enough to give any physically realistic function form of $D(\alpha)$.

Real surfaces allow the following restrictions on $D(\alpha)$:

(1) $D(\alpha)$ must exist for all values of α at least between 0 and $\pi/2$. Any realistic rough surface will have some microareas at any given value of α between 0 and $\pi/2$.

(2) $D(\alpha)$ must be finite for all values of α, since no real surface would contain any perfectly flat or perfectly cylindrical microarea.

(3) $D(\alpha)$ is positive for all values of α. This follows from Eq. (4), since σ_{zi} and $\sigma_{\alpha i}$ were defined positive.

(4) $D(\alpha)$ must be single valued. No physical quantity can have more than one value for itself at the same point.

(5) $D(\alpha)$ is continuous. Physical quantities never have perfect discontinuities.

Let $h(x)$ be a curve that when revolved about $x=0$ gives the surface of revolution (see Fig. 3). The relations $\rho_z=|x|$, $\rho_\alpha=[1+(h')^2]^{3/2}/|h''|$ (primes indicate derivatives with respect to x), and $\alpha=\tan^{-1}(-h')$ hold. Substitution into Eq. (6) gives

$$D[\tan^{-1}(-h')]=C|x|[1+(h')^2]^{3/2}/|h''| \ . \qquad (7)$$

We can show by elementary means that a solution $h(x)$ must exist for this differential equation for any functional form of $D(\alpha)$ [under the realistic constraints that $D(\alpha)$ exist and be continuous between $\alpha=0$ and $\pi/2$]. Thus there must exist a surface of revolution $h(x)$ capable of representing any realistic rough surface $D(\alpha)$.

Some limits to the form of $h(x)$ can be established such that $h(x)$ is still capable of giving any possible realistic form of $D(\alpha)$; that is, because of the nature of Eq. (7), certain options are allowed, the implementation of which will not limit the ability of $h(x)$ to give any $D(\alpha)$. Two options are the arbitrary choices of two boundary conditions for a second-order differential equation: one condition on $h'(x)$ and one on $h(x)$. This is allowed because a solution to a differential equation is still a solution when restricted by a boundary condition, and the set of solutions $h(x)$ for all forms of $D(\alpha)$ are still solutions for their respective forms of $D(\alpha)$ even though the same boundary condition is applied to every $h(x)$. A third option is the choice of a plus or minus sign for $h''(x)$. This is allowed because only its absolute value appears in Eq. (7).

Thus far, we have found two types of ways of limiting the functional form of $h(x)$ while not limiting its ability to give any physically realistic form of $D(\alpha)$. These consisted of three arbitrary restrictions on $h(x)$ that result from the nature of Eq. (7) and five restrictions on $D(\alpha)$ that result from real surfaces. We will apply these to determine a very limited type of curve for $h(x)$, a type that is still versatile enough to give any physically realistic form of $D(\alpha)$. First, let the boundary condition on $h'(x)$ be $h'(0)=0$. This makes the slope of $h(x)$ zero at $x=0$. Second, use the option on the sign of $h''(x)$ to make $h''(0)$ negative. This forces the slope of $h(x)$ to begin to decrease as x begins to increase (from zero). Third, the $h(x)$ cannot have inflection points. If it did, the same value of α would occur at different parts of the curve. Because D could have different values at these two points, there could be two values of D for the one value of α. Thus, D would be multiple valued, and restriction (4) would be violated. Fourth, $h(x)$ cannot be a straight line at any point. If it were, ρ_α would be infinite at that point, by Eq. (6) D would be infinite, and restriction (2) would be violated. The slope has been forced to be level (zero) at $x=0$ and to start to slope down (decrease) as x starts to increase. Because $h(x)$ can have no inflection points or straight-line segments, the slope must continue to decrease with increasing x. Thus the slope must become vertical $(-\infty)$ at a finite value of x (say, x_0). If the slope only approached vertical as x approached infinity, $h(x)$ would approach a straight line, D would approach infinity, and restriction (2) would be violated. Last, let the boundary condition on $h(x)$ be $h(x_0)=0$. This forces the vertically sloping part of $h(x)$ to lie on the x axis.

In summary, a limited functional form of $h(x)$ capable of giving any physically realistic functional form of $D(\alpha)$ resembles the curve in Fig. 3. This curve is level at $x=0$, decreases with increasing x without inflection points or straight sections, and becomes vertical as it crosses the x axis. Or, in terms of α, the curve is such that $\alpha = 0$ at $x = 0$ and α increases without stopping or turning back with increasing x to become $\alpha = 90°$ at $h = 0$.

It is instructive to visualize the random topography of a rough surface as equivalent to one large surface of revolution of very restricted shape, but what substantive contribution does this make? It does not put any limits on the functional form of the surface-structure function $D(\alpha)$, as we have just proved. But it may give insight into the discovery of better models to represent the surface structure. Indeed, as is shown later, the most obvious choice for a surface of revolution, an ellipsoid of revolution, gives a surface-structure function that appears better than the existing ones. Perhaps other shapes for the surface of revolution will give even better results. Also, maybe ones involving two or more parameters will give very good higher-order approximations. To derive the $D(\alpha)$ for any surface of revolution, we can follow the procedure used later for the ellipsoid.

Normalization

The unmeasurable variables α, z, and s and the jacobian J terms of the measurable variables β, θ, and ψ are

$$\tan z = \sin\psi \cos\theta / (\sin\beta + \cos\psi \cos\theta) \quad , \tag{8}$$

$$\tan\alpha = \sin\psi \cos\theta / [\sin z (\sin\theta + \cos\beta)] \quad , \tag{9}$$

$$\cos s = \cos z \sin\alpha \sin\beta + \cos\alpha \cos\beta \quad , \tag{10}$$

and

$$J\left(\frac{\alpha, z}{\theta, \psi}\right) = \frac{-\cos\theta}{4\cos s \sin\alpha} \quad . \tag{11}$$

Substituting Eqs. (11) and (6) into (5) gives

$$f_{rI}(\beta;\ \theta, \psi) = \mathcal{R}(s) \frac{1}{4\cos\beta} \frac{C\rho_\alpha(\alpha)\rho_z(\alpha)}{\sin\alpha} \quad . \tag{12}$$

As follows, the constant C can be found so as to make the expression for the reflected intensity-distribution function f_{rI} an absolute quantity. Because the average irregularity is a surface of revolution, the region around $\alpha = 0$ is approximately a spherical mirror surface, of radius $\rho_\alpha(0)$. For normal incidence and reflection, the BRIDF for this spherical mirror is equal to the BRIDF of our model, $f_{rI}(0; 90°, 0)$, and C is found from this equality.

The focal length of a spherical mirror is equal to half of the radius, $\frac{1}{2}\rho_\alpha(0)$, and collimated incident light reflects from it into a cone with the focal point as its vertex. A flux of $(\Phi_I/A_I)\Delta A$ is incident on a small area ΔA on the top of the mirror. If the single large surface of revolution is to replace all of the illuminated areas of the real surface, it must have a uniform flux density incident on all parts of it and have no radiation missing it. Therefore, the area of its base $\pi\rho_z^2(\pi/2)$ must equal the incident beam cross-sectional area A_I; therefore $\Phi_I \Delta A / [\pi\rho_z^2(\pi/2)]$ is the flux incident on ΔA. Upon reflection, this flux is attenuated by $\mathcal{R}(0)$ and is diverged into $\Delta A / [\frac{1}{2}\rho_\alpha(0)]^2$ steradians. Then $\mathcal{R}(0)$ multiplied by the incident flux and divided by this number of steradians gives the reflected intensity. Additionally dividing this by Φ_I gives the BRIDF for the spherical mirror,

$$f_{rI}(0; 90°, 0) = \mathcal{R}(0)\rho_\alpha^2(0) / [4\pi\rho_z^2(\pi/2)] \quad . \tag{13}$$

For the rough-surface model at normal incidence and reflection, $\beta = 0$, $\theta = \pi/2$, $\alpha = 0$, $s = 0$, and $\rho_z(0) = 0$ occur, and Eq. (12) becomes

$$f_{rI}(0; 90°, 0) = \tfrac{1}{4} C \mathcal{R}(0) \rho_\alpha(0) \lim_{\alpha \to 0} [\rho_z(\alpha)/\sin\alpha] \quad . \tag{14}$$

Equating Eqs. (13) and (14), solving for the normalization constant C, and substituting C into Eq. (12), we obtain the normalized or absolute BRIDF,

$$f_{rI}(\beta; \theta, \psi) = \mathcal{R}(s) \frac{1}{4\pi\cos\beta} \frac{\rho_\alpha(0)\rho_\alpha(\alpha)\rho_z(\alpha)}{\rho_z^2(\pi/2) \lim_{\alpha\to 0}[\rho_z(\alpha)/\sin\alpha]\sin\alpha} \quad . \tag{15}$$

Ellipsoid of revolution average-surface-irregularity approximation

A sphere is one possible model for the average surface irregularity, but it contains no parameter that can be varied to change the surface structure. An ellipsoid of revolution

$$h = e(1 - x^2)^{1/2} \quad , \tag{16}$$

has one such parameter e, the ratio of the length of the axis rotated about to the length of the axis rotated. As e decreases, the model irregularity becomes flatter and reflects more light in the specular direction. This is a useful property, because the wide variety of real surfaces contains a continuous distribution of diffuse to highly specular surfaces. Evaluating Eq. (15) for the $h(x)$ of Eq. (16)

$$-h' = \tan\alpha, \quad x = \tan\alpha(\tan^2\alpha + e^2)^{-1/2} = \rho_z(\alpha),$$

$$\rho_\alpha(\alpha) = [1 + (h')^2]^{3/2} / |h''| \quad ,$$

we obtain

$$f_{rI}(\beta;\ \theta, \psi) = \frac{\mathcal{R}(s,n,k)}{4\pi\cos\beta} \frac{e^2}{(e^2\cos^2\alpha + \sin^2\alpha)^2} \quad . \tag{17}$$

(Division by $\sin\theta$ gives the BRDF.)

EXPERIMENTAL

Earlier, we speculated that the concept of an average surface of revolution representing the surface structure might give insight into the choice of a more accurate and useful surface-structure function.

The first choice, the ellipsoid of revolution, resulted in the surface-structure function given by the last term in Eq. (17). Normalized to one at $\alpha = 0$, it is

$$e^4 / (e^2\cos^2\alpha + \sin^2\alpha)^2 \quad . \tag{18}$$

The microarea distribution function for the ellipsoid is easily shown to be proportional to $\rho_\alpha\rho_z/\sin\alpha$ and thus, by comparison with Eqs. (12) and (17), proportional to Eq. (18). Rense[12] found a way to determine the microarea distribution function from reflectance-distribution-function measurements. Figure 4 presents these data (for light) for one rough surface, along with plots of best fits for our structure function and four microarea distribution functions derived by applying Rense's method to four ray-reflectance-distribution functions found in the literature.

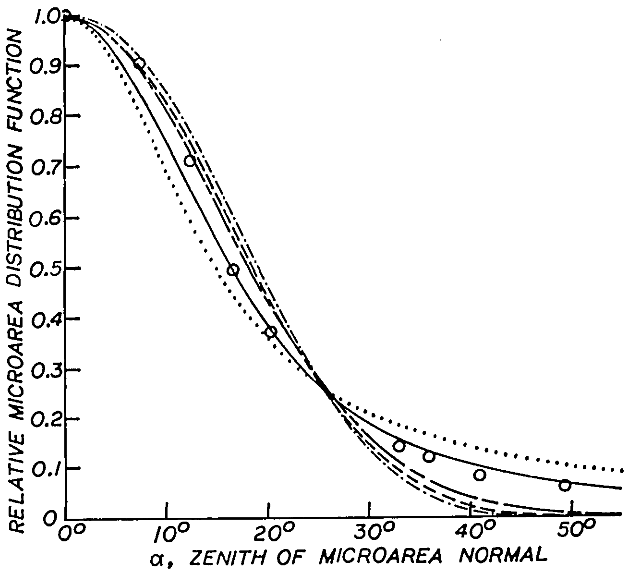

FIG. 4. Comparison of best fits of some surface-structure functions (equivalent to the microarea or facet distribution function). ○ Data taken from Rense (Ref. 12). ——— Plot of our function $e^4/(e^2\cos^2\alpha+\sin^2\alpha)^2$, $e=0.40$. ······· Plot of function $e^2/(e^2\cos^2\alpha+\sin^2\alpha)$, $e=0.25$, originated by Berry (Ref. 7). —·—·—· Plot of function $(\cos^{-4}\alpha)\exp(-A^2\tan^2\alpha)$, $A^2=7.62$, derived from Beckmann's (Ref. 18) result for a surface characterized by a gaussian distribution of surface heights and an autocorrelation length, in the ray approximation. ---- Plot of the function $(\cos^{-2}\alpha)\exp(-A^2\tan^2\alpha)$, $A^2=6.93$, used by Sirohi (Ref. 13), originated by Berry (Ref. 7). ——— Plot of an approximation $\exp(-A^2\alpha^2)$, $A^2=0.0021/\mathrm{deg}^2$) of --- used by Rense (Ref. 12) for small values of α.

The ellipsoid model may prove useful by allowing estimations of its parameter (e) to a reasonable accuracy simply from visual examination of a surface's microstructure. On each of the surfaces we examined, one of the authors has visually estimated the shape of the average ellipsoid by observing cross sections of surface irregularities and by observing variations of abundances of surface microareas with orientation relative to the macrosurface. (Magnification was sufficient so that most microareas appeared glossy. If it had not been, the determinate microstructure would have been at smaller size scales.) These visual estimations of e will be compared with optimized values. For each surface, relative BRIDF measurements (using 6328 Å light) were made for numerous combinations of values of the variables β, θ, ψ, and the two states of incident linear polarization. The model, combined with lambertian reflection, contains the parameters; e, n, $k=0$ for dielectrics, and ρ_L (β; 2π) (the directional-hemispherical reflectance[14,19] of the lambertian component). These were varied (optimized) until the best over-all fit to the measurements was obtained.

A dull-black optical antireflection paint appeared as a pile of approximately spherical globules. Its e was visually estimated to be ~0.8, and the optimized value was 0.89. The e of the surface of a sample of concrete cement was difficult to estimate but was between 0.5 and 1.0. Optimization gave $e=0.7$. The e for a sample of semiglossy dark paint was estimated to be between 0.05 and 0.1. Lightly soiled, it would be somewhat higher; optimization for a lightly soiled sample gave 0.2. The surface of a sample of wood consisted of circular cylinders. Such cylinders, randomly oriented, can be shown analytically to be approximately equivalent to an ellipsoid with $e=0.5$. Optimization gave $e=0.5$. A sample of grass sod consisted of semiglossy blades that were predominately vertical. This surface structure is too complex to permit an accurate estimation of e, but there is obviously much more microsurface area at large angles from the macrosurface than that characteristic of a sphere ($e=1$). We can only estimate e to be considerably greater than 1.0; the optimization gave $e=1.6$.

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